Maxwell's Equations and Einstein-Gravity in the Planck Aether Model of a Unified Field Theory

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In the Planck aether substratum model it is assumed that space is densely filled with both positive and negative Planck masses described by a nonlinear nonrelativistic operator field equation. Without expenditure of energy this substratum can form a lattice of vortex rings, with the vortex core radius equal the Planck length. The vortex ring radius is determined by a universal minimum energy quantum Reynolds number, making the ring radius and lattice spacing about $10^3 - 10^4$ times larger than the Planck length. The zero point fluctuations of the Planck masses bound in the vortex filaments become the source of virtual compression waves, which if quantized lead to Newton's law of gravitation, including the correct value for the gravitational constant. This scalar gravitational force couples the vortex rings, which thereby can transmit two types of transverse waves through the vortex lattice. The first type involves the tilting of the vortex rings and can be described by Maxwell's electromagnetic field equations. The second type involves the elliptic deformation of the rings and can be described by Einstein's gravitational field equations. Einstein gravity is therefore explained as resulting from the symmetric, and Maxwell's electromagnetism from the antisymmetric distortions of the Planck aether.

Special relativity follows as a dynamic symmetry for objects held together by electromagnetic forces, and general relativity if these objects are placed in a gravitational field. Both special and general relativity, though, turn out to be low energy approximations, breaking down near the Planck scale, eliminating all divergences and singularities.

Finally, the large difference between the electromagnetic and gravitational coupling constants is quantitatively explained to result from the negative masses in the Planck aether.

1. Introduction

In the Planck aether substratum model it is assumed that space is densely filled with an equal number of positive and negative Planck masses ($\pm m_p$) each occupying a volume determined by the Planck length r_p, and obeying an exactly nonrelativistic law of motion [1]. With the interactions between the Planck masses assumed to be contact type delta function potentials, this many body problem can be conveniently described by a Heisenberg-type nonlinear operator field equation for the field operators $\psi_\pm$, of the positive and negative Planck masses:

$$i\hbar\frac{\partial\psi_\pm}{\partial t} = \mp\frac{\hbar^2}{2m_p}\nabla^2\psi_\pm + 2\hbar c r_p^2(\psi_+^\dagger\psi_+ - \psi_-^\dagger\psi_-)\psi_\pm, \quad (1.1)$$

where $\psi_\pm, \psi_\pm^\dagger$ obey the canonical commutation relations

$$[\psi_\pm(\boldsymbol{r})\psi_\pm^\dagger(\boldsymbol{r}')] = \delta(\boldsymbol{r}-\boldsymbol{r}'),$$
$$[\psi_\pm(\boldsymbol{r})\psi_\pm(\boldsymbol{r}')] = [\psi_\pm^\dagger(\boldsymbol{r})\psi^\dagger(\boldsymbol{r}')] = 0. \quad (1.2)$$

Making the Hartree approximation

$$\langle\psi_\pm^\dagger\psi_\pm\psi_\pm\rangle \simeq \langle\psi_\pm^\dagger\rangle\langle\psi_\pm\rangle\langle\psi_\pm\rangle \quad (1.3)$$

one obtains from (1.1) a nonlinear Schrödinger equation of the type known from the Landau-Ginzburg theory of superfluidity:

$$i\hbar\frac{\partial\varphi_\pm}{\partial t} = \mp\frac{\hbar^2}{2m_p}\nabla^2\varphi_\pm + 2\hbar c r_p^2[\varphi_+^*\varphi_+ - \varphi_-^*\varphi_-]\varphi_\pm, \quad (1.4)$$

where

$$\varphi_\pm \equiv \langle\psi_\pm\rangle, \quad \varphi_\pm^* \equiv \langle\psi_\pm^\dagger\rangle. \quad (1.5)$$

Making the Madelung transformation

$$\begin{aligned} n_\pm &= \varphi_\pm^*\varphi_\pm, \\ n_\pm \boldsymbol{v}_\pm &= \mp\frac{i\hbar}{2m_p}[\varphi_\pm^*\nabla\varphi_\pm - \varphi_\pm\nabla\varphi_\pm^*], \\ V &= 2\hbar c r_p^2[n_+ - n_-], \\ Q_\pm &= \mp\frac{\hbar^2}{2m_p}\frac{\nabla^2\sqrt{n_\pm}}{\sqrt{n_\pm}}, \end{aligned} \quad (1.6)$$

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where V is the ordinary and $Q_\pm$ the quantum potential, (1.4) is reduced to

$$\frac{\partial n_\pm}{\partial t} + \operatorname{div}(n_\pm \boldsymbol{v}_\pm) = 0,$$
$$\frac{\partial \boldsymbol{v}_\pm}{\partial t} + (\boldsymbol{v}_\pm \cdot \nabla)\boldsymbol{v}_\pm = \mp \frac{1}{m_p} \operatorname{grad}(V + Q_\pm) \tag{1.7}$$

with

$$\varphi_\pm = A_\pm e^{iS_\pm}, \quad A_\pm \geqq 0, \quad 0 \leqq S_\pm < 2\pi. \tag{1.8}$$

One has

$$n_\pm = A_\pm^2,$$
$$\boldsymbol{v}_\pm = \pm \frac{\hbar}{m_p} \operatorname{grad} S_\pm, \tag{1.9}$$
$$S_\pm(\boldsymbol{r}, t) = \pm \frac{m_p}{\hbar} \int_{\boldsymbol{r}_0}^{\boldsymbol{r}} \boldsymbol{v}_\pm(\boldsymbol{r}', t)\, \mathrm{d}\boldsymbol{r}' + S_\pm^0(t).$$

Uniqueness of $\varphi_\pm$ requires that

$$\oint \boldsymbol{v}_\pm \cdot \mathrm{d}\boldsymbol{r} = \pm n h/m_p, \quad n = 0, 1, 2, \ldots. \tag{1.10}$$

From (1.10) follow quantized vortices of the form (for $n = 1$)

$$v_\varphi = c(r_p/r), \quad r > r_p; \quad v_\varphi = 0, \quad r < r_p \tag{1.11}$$

and the Helmholtz vortex theorem

$$\frac{\mathrm{d}}{\mathrm{d}t} \oint \boldsymbol{v}_\pm \cdot \mathrm{d}\boldsymbol{r} = 0, \tag{1.12}$$

which implies the persistence of these vortices in the superfluid frictionless Planck aether.

For the wave function one has

$$\varphi_\pm = |\sqrt{n_\pm}| \exp\left[\pm \frac{i m_p}{\hbar} \int \boldsymbol{v}_\pm \cdot \mathrm{d}\boldsymbol{r} + \frac{i}{\hbar} S_\pm^0(t)\right],$$
$$S_\pm^0(t) = \int_0^t E_\pm(t)\, \mathrm{d}t, \tag{1.13}$$
$$E_\pm(t) = \int (Q_\pm \pm \tfrac{1}{2} m_p v_\pm^2 + V) n_\pm\, \mathrm{d}\boldsymbol{r},$$

where $E_\pm$ is the total energy of the two superfluids.

2. The Groundstate

The symmetry of the Planck aether, consisting of two superfluids, one possessing positive and the other one negative mass, which are in relative mutual rest to each other, can be broken without the expenditure of energy by the formation of pairs of quantized vortices with opposite mass,with their positive and negative kinetic energy adding up to zero. Besides the distinguished state of total zero energy with both superfluids mutually at rest, there are many other states for which the energy is zero, but where the two superfluids form a tangle of positive and negative mass vortex filaments. According to the laws of vortex dynamics [2], a constant force acting on a vortex results in a constant velocity of the vortex. Applied to the tangle of vortex filaments, the mutual interaction of the vortices is bringing them into a state of mutual motion. As a result, they are going to suffer frequent collisions, through which the filaments are snapped and reconnected with other filaments. As computer simulations have shown, this process it ultimately leading to the formation of a lattice of vortex rings, with the ring radius r_0 roughly equal the distance of separation between neighboring vortex rings.

To determine this distance of separation one might minimize the energy $E_\pm$ using the wave function (1.13). To get a rough estimate for the ring radius r_0 (resp. lattice constant of the vortex lattice), we may use instead an analogy from classical hydrodynamics. There, the viscous drag has always a sharp minimum for a very large Reynolds number. For an object possessing cylindrical symmetry, one finds experimentally that Re $\simeq$ 250 000.

The energy dissipation caused by the viscous drag is determined by the equation

$$\frac{\partial \boldsymbol{v}}{\partial t} = \nu \nabla^2 \boldsymbol{v}, \tag{2.1}$$

where ν is the kinematic viscosity. A comparison with Schrödinger's equation

$$\frac{\partial \psi}{\partial t} = \frac{i\hbar}{2 m_p} \nabla^2 \psi \tag{2.2}$$

suggests that quantum mechanics can be seen to be governed by an imaginary viscosity. The drag in classical fluid dynamics implies a time dependent velocity field of the form

$$v \propto e^{-\gamma t}, \tag{2.3}$$

where $\gamma = \gamma_{\min}$ for Re $\simeq$ 250 000. Replacing the real with an imaginary viscosity, would by analytic continuation of (2.3) result in an expression of the form

$$v \propto e^{i\gamma t}, \tag{2.4}$$

and for the kinetic energy in a form

$$v^2 \propto e^{2i\gamma t} \propto E_\pm. \tag{2.5}$$

According to (2.5) the imaginary quantum viscosity should be equal to

$$\nu_Q = i\hbar/m_\mathrm{p} = i\,c\,r_p\,. \tag{2.6}$$

With this definition, the quantum Reynolds number would be

$$\mathrm{Re}^Q = i\,\frac{r\,v}{\nu_Q}\,. \tag{2.7}$$

At the surface of the vortex filament where $r\,v = r_\mathrm{p}\,c$ one has $\mathrm{Re}^Q = 1$. The quantum viscosity averaged over the space occupied by a linear vortex filament, separated from its neighboring filaments by the distance R, would then be $\bar{\nu}_Q = i\,c\,r_\mathrm{p}(r_\mathrm{p}/r_0)^2$, and the average quantum Reynolds number

$$\overline{\mathrm{Re}^Q} = i\,\frac{r\,v}{\bar{\nu}_Q} = \left(\frac{r_0}{r_\mathrm{p}}\right)^2. \tag{2.8}$$

Assuming that the value of the quantum Reynolds number for the minimum energy configuration is of the same order of magnitude as the classical Reynolds number Re = 250 000 for the minimum drag, one has*

$$r_0/r_\mathrm{p} \simeq 500\,. \tag{2.9}$$

The situation for a lattice of vortex rings is somewhat different, because through the interaction of vortex filaments with an opposite direction of circulation, their velocity is increased by the factor $\ln(8\,r_0/r_\mathrm{p})$ therefore, to keep $v_\varphi < c$ the separation of the vortices has to be increased, with the ratio r_0/r_p to be determined by the equation

$$(r_0/r_\mathrm{p})/\ln(8\,r_0/r_\mathrm{p}) \simeq 500\,, \tag{2.10}$$

which yields

$$r_0/r_\mathrm{p} \simeq 5000\,. \tag{2.11}$$

3. Compressional Waves

The most simple disturbances of the Planck aether are compressional waves, which in their quantized version are phonons. We consider them in the wave length range $r_\mathrm{p} \ll \lambda \ll r_0$. Because $|Q_\pm|/V \sim (r_\mathrm{p}/r_0)^2$ we can neglect $Q_\pm$ against V. For $\lambda \ll r_0$, we can furthermore neglect the effect the compression of the vortex ring lattice may have.

In the undisturbed Planck aether there are $n_\pm = 1/2\,r_\mathrm{p}^3$ Planck masses per unit volume. Superimposing a small disturbance on n_+ $(n' \ll n_+ \equiv n)$ with n_- remaining unchanged, leads to a velocity disturbance $\boldsymbol{v}'_+ \equiv \boldsymbol{v}$. For small disturbances (1.7) can be linearized:

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{2\hbar\,c\,r_\mathrm{p}^2}{m_\mathrm{p}}\nabla n'\,, \tag{3.1}$$

$$\frac{\partial n'}{\partial t} + n\,\mathrm{div}\,\boldsymbol{v} = 0\,. \tag{3.2}$$

With $\Theta \equiv \mathrm{div}\,\boldsymbol{v}$ one obtains from (3.1) and (3.2) the wave equation

$$-(1/c^2)\,\partial^2\Theta/\partial t^2 + \nabla^2\Theta = 0\,. \tag{3.3}$$

The zero point fluctuations of the Planck masses bound in the vortex filaments excite virtual compression waves which after quantization lead to an attractive inverse square law force for a point source. To compute the coupling constant of this interaction we note that the average kinetic energy of the Planck masses confined in the radial and azimuthal dimension of the vortex filament is

$$\Delta E = \mp(2/3)\overline{(1/2)\,m_\mathrm{p}\,v^2} = \mp(1/6)\,\hbar\,c/r_\mathrm{p}\,. \tag{3.4}$$

Because the vortex core is hollow, this energy must be counted negative for the positive mass component of the Planck aether and positive for its negative mass component. With a volume $\pi r_\mathrm{p}^2 \cdot r_\mathrm{p} = \pi r_\mathrm{p}^3$ occupied by each Planck mass within the vortex filament, the value for the kinetic energy density ε_k is

$$\varepsilon_k = \mp(1/6\,\pi)\,\hbar\,c/r_\mathrm{p}^4\,. \tag{3.5}$$

Assuming Newton's law of gravity, the gravitational field in the vortex core would be

$$F = \mp 2\sqrt{G}\,(m_\mathrm{p}/r_\mathrm{p}^3)\,r \tag{3.6}$$

* In classical viscous gas dynamics the minimum radius $r_{\min}$ of a vortex core must be of the order of a mean free path λ, with the kinematic viscosity of the order $\nu \sim \lambda\,v_\mathrm{th}$, where v_th is the thermal particle velocity. With the maximum velocity reached at the vortex core equal to v_th, The Reynolds number at the surface of the core is $\mathrm{Re} = v\,r/\nu = v_\mathrm{th}\,\lambda/\nu = 1$. Therefore, with the same argument used for the quantum Reynolds number, the viscosity averaged over a lattice volume occupied by a linear vortex filament, would be $\bar{\nu} = \lambda\,v_\mathrm{th}\,(\lambda/r_0)^2$ and hence the average Reynolds number $\overline{\mathrm{Re}} = r\,v/\bar{\nu} = \lambda\,v_\mathrm{th}/\bar{\nu} = (r_0/\lambda)^2$. For Re = 250 000, the distance of separation r_0 would then be several 100 times larger than the core radius. Not only is this result in surprisingly good agreement with a detailed theoretical analysis of the minimum drag problem or the Karman vortex street carried out by Schlayer [3], but is also experimentally confirmed. An experimental value for the universal quantum Reynolds number in the case of quantized vortices could possibly be obtained by experiments with superfluid helium [4].

with an average energy density

$$\varepsilon_g = \mp \overline{F^2/8\pi} = \mp (1/6\pi)\hbar c/r_p^4, \tag{3.7}$$

which is the same as (3.4). The quantized compression waves therefore lead to a scalar Newtonian law of gravity, with the correct value of the gravitational coupling constant, solely resulting from the zero point fluctuations of the Planck masses bound in the vortex filaments.

The Planck masses distributed along the vortex ring lead to a modification of the compression waves for wavelengths near $\lambda \sim r_0$. The number of Planck masses per unit volume attached to the vortex rings of radius r_0 in a vortex lattice with a lattice constant $\sim r_0$, is $\sim (r_p/r_0)^2 n$, where $n = 1/2 r_p^3$. The Planck masses, therefore give rise to a force field $\boldsymbol{F}$ determined by the Poisson-type equation

$$\operatorname{div} \boldsymbol{F} = 4\pi \sqrt{2\hbar c}\,(r_p/r_0)^2 n'. \tag{3.8}$$

The force is here repulsive, because an increased aether density displaces and, thereby, dilutes the density of the vortex filaments embedded in it. With (3.8), (3.1) is modified as follows:

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{2\hbar c r_p^2}{m_p} \nabla n' + \frac{\sqrt{2\hbar c}}{m_p} \boldsymbol{F}. \tag{3.9}$$

The continuity equation (3.2) is unchanged. With $\Theta \equiv \operatorname{div} \boldsymbol{v}$ one then obtains the modified wave equation

$$-(1/c^2)\,\partial^2\Theta/\partial t^2 + \nabla^2\Theta - (\omega_0^2/c^2)\,\Theta = 0, \tag{3.10}$$

where

$$\omega_0^2 = 4\pi (c/r_0)^2. \tag{3.11}$$

The wave equation (3.10) has the dispersion relation

$$\omega^2 = c^2 k^2 + \omega_0^2 \tag{3.12}$$

with a phase velocity

$$v_{ph} = \frac{\omega}{k} = \frac{c}{\sqrt{1 - (\omega_0/\omega)^2}} \tag{3.13}$$

and a group velocity

$$v_{gr} = c\sqrt{1 - (\omega_0/\omega)^2}, \tag{3.14}$$

such that $v_{ph} v_{gr} = c^2$. It follows from (3.13) that the compression waves have a cut-off at the wavelength $\lambda \sim r_0$, restricting them to wavelengths $r_p < \lambda < r_0$. The scalar Newtonian law of gravity is for this reason restricted to the range of energies $\hbar c/r_0 < \hbar c/r_p = m_p c^2$.

4. Transverse Waves

For wave lengths $\lambda \gg r_0$ there are two kinds of transverse waves. Unlike an elastic body, where transverse waves can only be the result of a strain, in a vortex lattice waves associated with a rigid rotational deformation are also possible. These waves result from the rotational fluid flow of a vortex, which gives the flow a stiffness against a change in its direction in space, very much as it is the case for a spinning top. For a lattice of vortex rings these waves result in a direction-changing motion of the vortex rings. As first shown by Kelvin [5] a vortex lattice can for this reason propagate a transverse wave, which for small amplitudes has the same property as Maxwell's electromagnetic wave. It was shown by Kelly [6] that the existence of these waves is quite insensitive to the chosen form of the vortex lattice, and that they are even possible in a vortex sponge of randomly distributed vortex filaments.

The other kind of wave which can propagate through a vortex lattice is similar to a transverse wave in an elastic body and is associated with an elliptic deformation of the vortex rings. It will be shown that these waves can be identified with Einstein's gravitational waves.

According to Helmholtz [7] the most general displacement of a deformable body consists of 1) a translation, 2) a rotation, and 3) a strain. A wave associated with a translational displacement is not known**. In a solid only the displacement by a strain leads to a disturbance propagated as a wave, because only a strain generates there a stress acting against its deformation. In a vortex sponge, a rotational displacement can, in addition, lead to a wave, because of the rotational-inertial stiffness counteracting such displacements. The strange difference in character between the electromagnetic and the gravitational fields has in the Planck aether model therefore an almost trivial explanation because, according to the theorem by Helmholtz, a disturbance of a body can, in general, always be decomposed into two irreducible parts of a symmetric and antisymmetric tensor. This decomposition alone, however, does not explain the two other important properties distinguishing gravity from electromagnetism: 1) the nonlinearity of the gravitational

** With the exception of Alfvén waves, but they are only possible in a magnetized plasma.

field equations and 2) the smallness of the gravitational coupling constant if compared with its electromagnetic counterpart. As will be shown below, the Planck aether model can even explain these properties.

To analyze the transverse waves, let $\boldsymbol{v} = \{v_x, v_y, v_z\}$ be the undisturbed velocity of the Planck aether and $\boldsymbol{u} = \{u_x, u_y, u_z\}$ a small superimposed velocity disturbance. Furthermore, let us take only those solutions for which $\operatorname{div} \boldsymbol{v} = \operatorname{div} \boldsymbol{u} = 0$. This means that we are only considering waves with no density disturbance. To reduce the problem to the solution of a differential equation we must go to the continuum limit. This can be done by letting the vortex lattice go from r_0 to r_p, and where r_p can be chosen arbitrarily small. The x-component of the equation of motion for a disturbance $\boldsymbol{u}$ is

$$\frac{\partial v_x}{\partial t} + \frac{\partial u_x}{\partial t} = -(v_x + u_x)\frac{\partial (v_x + u_x)}{\partial x} - (v_y + u_y)\frac{\partial (v_x + u_x)}{\partial y} - (v_z + u_z)\frac{\partial (v_x + u_x)}{\partial z} - \frac{1}{\varrho}\frac{\partial p}{\partial x}. \tag{4.1}$$

From the continuity equation $\operatorname{div} v = 0$ we have

$$v_x \frac{\partial v_x}{\partial x} + v_x \frac{\partial v_y}{\partial y} + v_x \frac{\partial v_z}{\partial z} = 0. \tag{4.2}$$

Subtracting (4.2) from (4.1) and taking the y-z average we find

$$\frac{\partial u_x}{\partial t} = -\frac{\partial (\overline{v_y v_x})}{\partial y} - \frac{\partial (\overline{v_z v_x})}{\partial z}, \tag{4.2 a}$$

and similarly, by taking the x-z and x-y averages:

$$\frac{\partial u_y}{\partial t} = -\frac{\partial (\overline{v_x v_y})}{\partial x} - \frac{\partial (\overline{v_z v_y})}{\partial z}, \tag{4.2 b}$$

$$\frac{\partial u_z}{\partial t} = -\frac{\partial (\overline{v_x v_z})}{\partial x} - \frac{\partial (\overline{v_y v_z})}{\partial y}. \tag{4.2 c}$$

We note that $\overline{v_x v_y} \neq \overline{v_y v_x}$, because for $\overline{v_x v_y}$ we took the x-z average, whereas, for $\overline{v_y v_x}$ the y-z average was taken. In general $\overline{v_i v_k} \neq \overline{v_k v_i}$. With the condition $\operatorname{div} \boldsymbol{u} = 0$ we obtain from (4.2 a–c)

$$\overline{v_i v_k} = -\overline{v_k v_i}. \tag{4.3}$$

Taking the x-component of the equation of motion, multiplying it by v_y and then taking the y-z average, and the y-component multiplied by v_x and taking the x-z average, finally subtracting the first from the second equation we find

$$\frac{\partial}{\partial t}(\overline{v_x v_y}) = -v^2\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right), \tag{4.4}$$

where $v^2 = \overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$ is the average microvelocity of the vortex lattice. Putting $\varphi_z = -\overline{v_x v_y}/2v^2$, (4.4) is just the z-component of

$$\frac{\partial \boldsymbol{\varphi}}{\partial t} = \frac{1}{2}\operatorname{curl} \boldsymbol{u}, \tag{4.5}$$

where $\varphi_x = -\overline{v_y v_z}/2v^2$, $\varphi_y = -\overline{v_z v_x}/2v^2$. Equations (4.2 a–c) then take the form

$$\frac{\partial \boldsymbol{u}}{\partial t} = -2v^2 \operatorname{curl} \boldsymbol{\varphi}. \tag{4.6}$$

Elimination of $\boldsymbol{\varphi}$ from (4.5) and (4.6) results in a wave equation for $\boldsymbol{u}$:

$$-(1/v^2)\,\partial^2 \boldsymbol{u}/\partial t^2 + \nabla^2 \boldsymbol{u} = 0. \tag{4.7}$$

In the collapsed vortex lattice, making the transition $r_0 \to r_p$, one should have for the microvelocity $v^2 = c^2$. In this limit, (4.7) would describe a transverse wave propagating with the velocity of light c. In reality though, $r_0 \gtrsim 10^3 r_p$, which means that the equation describing this wave would break down at an energy corresponding to the scale r_0. This energy is of the order 10^{16} GeV, which coincides with the energy of the grand unification scale obtained from the extrapolation of the experimentally measured running coupling constants to ultrahigh energies.

With $v = \mathrm{c}$ and putting $\boldsymbol{u} = \boldsymbol{E}$ and $\boldsymbol{\varphi} = -(1/2c)\boldsymbol{H}$, (4.5) and (4.6) have the same form as the two Maxwell vacuum field equations

$$-\frac{1}{\mathrm{c}}\frac{\partial \boldsymbol{H}}{\partial t} = \operatorname{curl} \boldsymbol{E}, \tag{4.8}$$

$$\frac{1}{\mathrm{c}}\frac{\partial \boldsymbol{E}}{\partial t} = \operatorname{curl} \boldsymbol{H}. \tag{4.9}$$

Adding (4.2) to (4.1) and taking the average over x, y and z, we have

$$\frac{\partial u_x}{\partial t} = -\frac{\partial \overline{v_x^2}}{\partial x} - \frac{\partial \overline{v_x v_y}}{\partial y} - \frac{\partial \overline{v_x v_z}}{\partial z}, \tag{4.10 a}$$

and similarly

$$\frac{\partial u_y}{\partial t} = -\frac{\partial \overline{v_y^2}}{\partial y} - \frac{\partial \overline{v_y v_z}}{\partial z} - \frac{\partial \overline{v_y v_x}}{\partial x}, \tag{4.10 b}$$

$$\frac{\partial u_z}{\partial t} = -\frac{\partial \overline{v_z^2}}{\partial z} - \frac{\partial \overline{v_z v_x}}{\partial x} - \frac{\partial \overline{v_z v_y}}{\partial y}. \tag{4.10 c}$$

Combining (4.10a–c) with the condition div $\boldsymbol{u}$ leads to

$$\frac{\partial^2}{\partial x_i \partial x_k}(\overline{v_i v_k}) = 0. \tag{4.11}$$

and for (4.10a–c) we can write

$$\frac{\partial u_k}{\partial t} = -\frac{\partial}{\partial x_i}(\overline{v_i v_k}). \tag{4.12}$$

Multiplying the v_i component of the equation of motion with v_k, and vice versa, its v_k component with v_i, adding both and taking the average, we find

$$\frac{\partial}{\partial t}(\overline{v_i v_k}) = -v^2\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right). \tag{4.13}$$

From (4.12) we have

$$\frac{\partial^2 u_k}{\partial t^2} = -\frac{\partial}{\partial t\, \partial x_i}(\overline{v_i v_k}), \tag{4.14}$$

and from (4.13)

$$\frac{\partial}{\partial x_i \partial t}(\overline{v_i v_k}) = -v^2\left(\frac{\partial}{\partial x_k}\frac{\partial u_i}{\partial x_i} + \frac{\partial^2 u_k}{\partial x_i^2}\right) = -v^2\frac{\partial^2 u_k}{\partial x_i^2}, \tag{4.15}$$

the latter because of div $\boldsymbol{u} = 0$. Eliminating $\overline{v_i v_k}$ from (4.14) and (4.15) and putting as before $v^2 = c^2$, finally results in

$$\frac{\partial^2 u_k}{\partial t^2} = c^2\frac{\partial^2 u_k}{\partial x_i^2} \tag{4.16}$$

or

$$\nabla^2 \boldsymbol{u} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{u}}{\partial t^2} = 0. \tag{4.17}$$

We claim that (4.17) can be interpreted as a linearized gravitational wave equation derived from Einstein's gravitational field theory. To demonstrate this equivalence, we are considering a gravitational wave propagating in the x-direction. It is described by the line element [8]

$$ds^2 = ds_0^2 + h_{22}\,dx_2^2 + 2\,h_{23}\,dx_2\,dx_3 + h_{33}\,dx_3^2, \tag{4.18}$$

where

$$h_{22} = -h_{33} = f(t - x/c),\qquad h_{23} = g(t - x/c) \tag{4.19}$$

with f and g two arbitrary functions, and ds_0^2 the line element in the absence of a gravitational wave. A deformation of an elastic body can likewise be described by a distorted line element as [9]

$$ds^2 = ds_0^2 + 2\,\varepsilon_{ik}\,dx_i\,dx_k, \tag{4.20}$$

where

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial \varepsilon_i}{\partial x_k} + \frac{\partial \varepsilon_k}{\partial x_i}\right). \tag{4.21}$$

In (4.21) $\boldsymbol{\varepsilon} = (\varepsilon_x, \varepsilon_y, \varepsilon_x)$ is the displacement vector, which is related to the velocity disturbance vector $\boldsymbol{u}$ by

$$\boldsymbol{u} = \frac{\partial \boldsymbol{\varepsilon}}{\partial t}. \tag{4.22}$$

In an elastic medium, transverse waves obey the wave equation

$$\nabla^2 \boldsymbol{\varepsilon} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{\varepsilon}}{\partial t^2} = 0, \tag{4.23}$$

which because of (4.22) is the same as (4.17). From the condition div $\boldsymbol{u} = 0$ and (4.22) then also follows div $\boldsymbol{\varepsilon} = 0$.

For a transverse wave propagating into the x-direction, $\varepsilon_x = \varepsilon_1 = 0$ and the condition div $\boldsymbol{\varepsilon} = 0$ there leads to

$$\frac{\partial \varepsilon_2}{\partial x_2} + \frac{\partial \varepsilon_3}{\partial x_3} = \varepsilon_{22} + \varepsilon_{33} = 0. \tag{4.24}$$

Hence

$$\varepsilon_{33} = -\varepsilon_{22}. \tag{4.25}$$

The identity with a gravitational wave is complete if one puts

$$2\,\varepsilon_{22} = h_{22},\qquad 2\,\varepsilon_{33} = h_{33},\qquad 2\,\varepsilon_{23} = h_{23}. \tag{4.26}$$

Figure 1 illustrates how an electromagnetic and a gravitational wave are distorting the vortex lattice.

5. Lorentz Invariance as a Dynamic Symmetry and Selection Principle

If solid bodies are held together by electromagnetic forces, or forces acting like them, Lorentz invariance can be explained as a dynamic symmetry. In the framework of the Planck aether model, the strong and weak force can be understood by a vortex substruc-

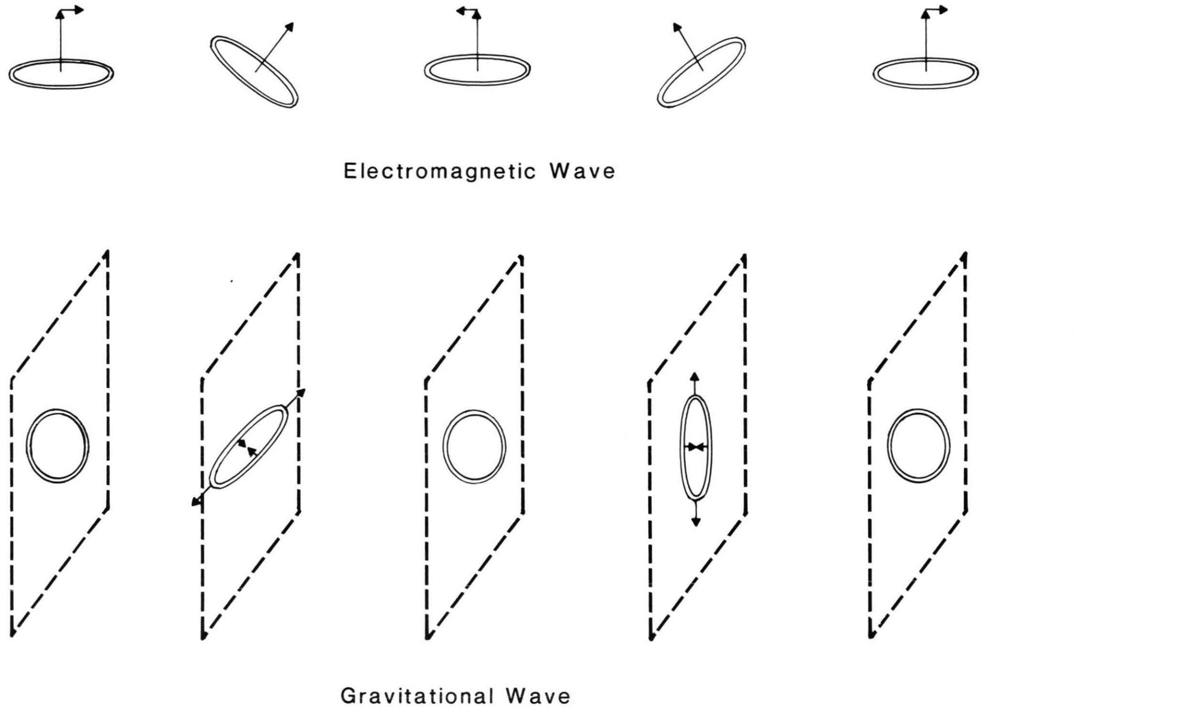

Fig. 1. The distortion of the ring vortices embedded in the Planck aether for an electromagnetic and gravitational wave.

ture very much like in the quantum Hall effect [10]. That the strong and weak force may thereby be derived from the electromagnetic force alone, is qualitatively supported by the fact that the smallest possible wavelength for the vortex waves is of the order of the vortex ring radius r_0, which happens to be the scale at which the coupling constants of the strong, weak and electromagnetic interactions become equal.

To show how Lorentz invariance can be seen as a derived dynamic symmetry, we consider a body at rest in the Planck aether, assumed to be in a state of internal static equilibrium. The scalar and vector potentials within the body are given as by Maxwell's equations [11]

$$\nabla^2 \Phi = -4\pi \varrho_e, \quad \nabla^2 \boldsymbol{A} = -(4\pi/c)\boldsymbol{j}_e, \tag{5.1}$$

supplemented by the gauge condition

$$\operatorname{div} \boldsymbol{A} = 0, \tag{5.2}$$

The charge and current densities ϱ_e and j_e, have their source in the body. If the body is accelerated to the constant velocity $\boldsymbol{u}$ against the Planck aether, the equations for Φ and $\boldsymbol{A}$ in a co-moving reference system under the Galilei-transformation

$$r' = \boldsymbol{r} - \boldsymbol{u}t, \quad t' = t \tag{5.3}$$

are:

$$\begin{aligned} \left[-\frac{1}{c^2}\left(\frac{\partial}{\partial t} - \boldsymbol{u}\cdot\nabla'\right)^2 + \nabla'^2\right]\Phi' &= -4\pi\varrho'_e, \\ \left[-\frac{1}{c^2}\left(\frac{\partial}{\partial t} - \boldsymbol{u}\cdot\nabla'\right)^2 + \nabla'^2\right]\boldsymbol{A}' &= -(4\pi/c)\boldsymbol{j}', \end{aligned} \tag{5.4}$$

where ∇' is the Galilei transformed Nabla operator. The Galilei invariance of the continuity equation $\partial\varrho'_e/\partial t + \operatorname{div}\boldsymbol{j}'_e = 0$ follows then directly from (5.4), provided the Lorentz gauge is made Galilei invariant:

$$\frac{1}{c}\frac{\partial\Phi'}{\partial t} + \operatorname{div}\boldsymbol{A}' = 0. \tag{5.5}$$

After the body has settled down to a new equilibrium state, for which $\partial/\partial t = 0$, one has

$$\begin{aligned} \left[\left(1 - \frac{u^2}{c^2}\right)\nabla'^2_{\parallel} + \nabla'^2_{\perp}\right]\Phi' &= -4\pi\varrho'_e, \\ \left[\left(1 - \frac{u^2}{c^2}\right)\nabla'^2_{\parallel} + \nabla'^2_{\perp}\right]\boldsymbol{A}' &= -(4\pi/c)\boldsymbol{j}'_e \end{aligned} \tag{5.6}$$

and

$$\operatorname{div} \boldsymbol{A}' = 0\,, \tag{5.7}$$

where $\nabla_{\parallel}$ and $\nabla_{\perp}$ are the components of the Nabla operator parallel and perpendicular to the direction of $\boldsymbol{u}$.

Comparing Eqs. (5.6), (5.7) with (5.1), (5.2), one immediately sees that (5.6), (5.7) are the same as (5.1), (5.2), if one puts everywhere, including in the sources,

$$\mathrm{d}\boldsymbol{r}'_{\parallel} = \sqrt{1 - u^2/c^2}\,\mathrm{d}\boldsymbol{r}_{\parallel}\,, \quad \mathrm{d}\boldsymbol{r}'_{\perp} = \mathrm{d}\boldsymbol{r}_{\perp}\,, \tag{5.8}$$

which makes

$$\left(1 - \frac{u^2}{c^2}\right) \nabla'^2_{\parallel} + \nabla'^2_{\perp} = \nabla^2\,, \tag{5.9}$$

Therefore, $\Phi' = \Phi$ and $\boldsymbol{A}' = \boldsymbol{A}$, provided $\varrho'_{\mathrm{e}}(\boldsymbol{r}') = \varrho_{\mathrm{e}}(\boldsymbol{r})$, $\boldsymbol{j}'_{\mathrm{e}}(\boldsymbol{r}') = \boldsymbol{j}_{\mathrm{e}}(\boldsymbol{r})$, implying a uniform contraction of the sources by the Fitzgerald-Lorentz factor $\sqrt{1 - u^2/c^2}$. The continuity and gauge conditions are unaffected by this change.

If all the interactions holding the body together behave like the electromagnetic interactions, all clocks should behave like light clocks, and in considering the combined effect of the Lorentz contraction and anisotropic light propagation in a moving frame makes a light clock move slower by the same factor $\sqrt{1 - u^2/c^2}$, as in special relativity [12]. The Lorentz contraction alone is, therefore, sufficient to derive the Lorentz transformations as a dynamic symmetry for objects in a state of internal equilibrium.

From the Lorentz transformation follows the relativistic addition theorem of velocities and from the velocity addition theorem the relativistic variation of the mass with the velocity, $m = m_0/\sqrt{1 - u^2/c^2}$. In conjunction with Newton's equation of motion $(\mathrm{d}t/\mathrm{d}t)(m\,\boldsymbol{v}) = \text{force}$, then follows the relativistic expression for the energy E. If all energy is electromagnetic in origin, one has $E = E_0/\sqrt{1 - u^2/c^2}$, where E_0 is the electromagnetic energy in the aether rest frame. From the Lorentz transformations and the relativistic expression for the energy follows the relativistically invariant Hamilton function. If translated into quantum mechanics it leads to the relativistically invariant Hamiltonian and ensures that the contraction effect does not change the pressure of the zero point energy. The pressure by the zero point energy results from the uncertainty principle, which for electromagnetic energy $m c^2$ is $\Delta m c^2 \cdot \Delta r \gtrsim \hbar$. If $\Delta r_{\parallel}$ changes into $\Delta r'_{\parallel} = \sqrt{1 - u^2/c^2\,\Delta r}$, it changes $\Delta m c^2$ into $\Delta m c^{2\prime} = \Delta m c^2/\sqrt{1 - u^2/c^2}$, and therefore keeps $(\Delta m c^2)'\,\Delta r'_{\parallel} = (\Delta m c^2)\,\Delta r_{\parallel}$ invariant under this change of scale.

In the Planck aether hypothesis Lorentz invariance as a dynamic symmetry would break down at the Planck length. With the Planck length being proportional to the square root of the gravitational constant, this means that in the limit where this constant is set equal to zero, the dynamic and kinematic interpretation of Lorentz invariance would give the same results for all energies and, therefore, would become indistinguishable*.

Because it is valid only for those objects which are in a state of static equilibrium, Lorentz invariance as a dynamic symmetry at the same time also acts as a dynamic selection principle. It would for this reason be violated for times shorter than what is required to establish a static equilibrium. If fermions are made up from objects having the dimension r_0, fermions with a lifetime

$$\tau \gg \tau_0 \sim r_0/c \sim 10^{-41}\,\mathrm{sec} \tag{5.10}$$

are sufficiently stable to satisfy Lorentz invariance as a dynamic selection principle. This is certainly true for all know fermions.

If Lorentz invariance must be understood as a dynamic symmetry valid only for times large compared to the time τ_0, it would mean that theorems like the CPT theorem, or the spin-statistics theorem derived under the assumption that special relativity is exactly fulfilled, could be violated for times shorter than this time. The dynamic selection principle would also exclude all those theories which are nonrenormalizable, because these theories depend on a cut-off at the Compton wavelength of some elementary particle, rather than a cut-off at $r_0 \sim 10^{-30}$ cm.

Relativistic invariance as a dynamic selection principle also leads to the minimum coupling principle which, however, would be valid only in the asymptotic limit of low energies. For example the relativistically

* In this limit the Planck mass would be infinite, with no quantum fluctuations of the Planck aether, and the positive and negative masses of the Planck aether would cancel each other out exactly. For a finite, albeit small, gravitational constant, the quantum fluctuations of the Planck aether would make Lorentz invariance exactly true only in the time and space average, increasingly violating this invariance in approaching the Planck length r_{p} and Planck time r_{p}/c. Because the quantum fluctuations disappear in the limit of an infinite Planck mass resp. vanishing Planck length, where both the kinematic and dynamic interpretation give identical results, one may say that Einstein's kinematic interpretation corresponds to a massless aether, reached in the limit $G \to 0$.

invariant Lagrangian L for Dirac spinors interacting with an electromagnetic field, should in general have the form of an infinite series [13]:

$$L = -\bar{\psi}\left(\gamma^\mu \frac{\partial}{\partial x^\mu} + m\right)\psi - \frac{1}{4}\left(\frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}\right)^2 + i e_0 A_\mu \bar{\psi}\gamma^\mu \psi + e_1\left(\frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}\right)\bar{\psi}\sigma^{\mu\nu}\psi + e_2 \bar{\psi}\psi\bar{\psi}\psi + \dots . \tag{5.11}$$

The units in (5.11) are chosen to make L integrated over space-time nondimensional. With this choice of units, e_0 is nondimensional, but e_1 has the dimension (length)^1 and e_2 the dimension (length)^2. Higher terms not written down have coupling constants expressed through higher powers of length. Since in our model the only length associated with electromagnetic waves is the vortex ring radius r_0, all the coupling constants $e_1, e_2, e_3 \dots$ should be of the form

$$e_n = a_n r_0^n \tag{5.12}$$

with the constants a_n of the order unity. The smallness of r_0, if compared with the electron Compton wave length, explains why one can neglect all the higher terms in the Lagrangian for energies which are small compared with the energy $\hbar c/r_0 \sim 10^{16}$ GeV. Only the term multiplied with e_0 and which remains in the asymptotic limit of low energies satisfies the minimum coupling principle. Since there is no reason why the higher terms should be excluded, the extremely good agreement of QED with the experiments done at low energies is from this perspective much not too impressive.

6. Maxwell's Equations

Whereas the compressional waves had their sources in the zero point fluctuations of the Planck masses bound in the vortex filaments, the electromagnetic charges would have to be explained by the zero point fluctuations involving a tilting motion of the vortex rings. The totality of the vortex rings can be seen as a fluid obeying an exactly nonrelativistic equation of motion. It therefore satisfies a nonrelativistic continuity equation which has the same form as the equation for charge conservation:

$$\frac{\partial \varrho_e}{\partial t} + \operatorname{div} \boldsymbol{j}_e = 0, \tag{6.1}$$

where ϱ_e and $\boldsymbol{j}_e$ are the electric charge and current density. Because the charges are the source of the electromagnetic field, one has

$$\operatorname{div} \boldsymbol{E} = 4\pi \varrho_e . \tag{6.2}$$

Therefore, in order to satisfy (6.1), a term must be added to the Maxwell equation (4.9) if charges are present. It thereby becomes

$$\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{4\pi}{c}\boldsymbol{j}_e = \operatorname{curl} \boldsymbol{H} . \tag{6.3}$$

The other Maxwell equation (4.8) is purely kinematic, as its substratum form (4.5) shows, and, therefore, is unchanged. Finally, because of (4.5) $\operatorname{div} \boldsymbol{\varphi} = 0$, it follows that

$$\operatorname{div} \boldsymbol{H} = 0 . \tag{6.4}$$

This completes the proof that Maxwell's equations can be derived from the Planck aether substratum model.

7. Einstein's Gravitational Field Equations

Because of (4.17), (4.21), (4.22) and (4.26), a gravitational wave propagating into the x-direction obeys the equation

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2}\right)h_{ik} = 0 \quad i,k = 1,2,3,4 . \tag{7.1}$$

By a space-time coordinate transformation this wave equation can be brought into the form [8]

$$\Box \psi_i^k = 0, \quad i,k = 1,2,3,4 \tag{7.2}$$

with the subsidiary (gauge) condition

$$\frac{\partial \psi_i^k}{\partial x^k} = 0 \tag{7.3}$$

and where the tensor ψ_i^k represents the gravitational field. According to the minimum coupling principle, which was shown to be valid in the low energy limit, the gravitational field equation in the presence of matter must have the form

$$\Box \psi_i^k = \varkappa \Theta_i^k, \quad \varkappa = \text{const}, \tag{7.4}$$

where Θ_i^k is a four-dimensional, relativistically invariant symmetric tensor. Because of (7.3), it obeys the equation

$$\frac{\partial \Theta_i^k}{\partial x^k} = 0 . \tag{7.5}$$

The only physically possible invariant tensor obeying Lorentz invariance as a dynamic selection principle and satisfying the conservation law (7.5) is the total energy momentum tensor of matter and gravitational field. As it was shown by Gupta [14], splitting Θ_i^k in its matter part T_i^k and gravitational field part t_i^k,

$$\Theta_i^k = T_i^k + t_i^k, \tag{7.6}$$

the field equation (7.4) can be brought into Einstein's form

$$R_{ik} - \tfrac{1}{2} g_{ik} R = \varkappa T_{ik}, \tag{7.7}$$

where it is expressed as a field equation in a non-euclidean space-time manyfold. According to our model, where space is euclidean and time absolute, with special relativity caused by true physical deformations, the reason why the field equation can be formulated by a noneuclidean manifold has nothing to do with a supposedly curved space-time. It rather results from the principle of equivalence, whereby all bodies, given the same initial conditions, follow the same trajectory. As in the dynamic interpretation of special relativity, where the Minkowskian space-time manyfold is seen as an illusion caused by true deformations of bodies, a Riemannian manyfold must here be seen as an illusion as well, caused by the same true physical deformations in conjunction with the illusion played through the nonlinearity of the field equations.

The unphysical feature of (7.7), that its solution always lead to space-time singularities, is resolved by recognizing that (7.7) is valid only in a low-energy limit. The field equations (7.7) are obtained from Hilbert's action

$$S = \int (R + \varkappa T) \sqrt{-g}\, \mathrm{d}\Omega \tag{7.8}$$

with the requirement that

$$\delta S = 0 \tag{7.9}$$

Together with (7.8) this requirement is the cause for the singular solutions of (7.7), because to make $\delta S = 0$ leads to $R \to \infty$ for $T \to \infty$. Under conditions of gravitational collapse $T \to \infty$ and hence with necessity $R \to \infty$. As the general QED Lagrangian (5.11) should include higher order terms with coupling constants of the dimensions r_0^n, the same should be true for the generalized gravitational Lagrangian. For example, it might be of the form

$$L_g = \sqrt{-g}\, [R + a_1 r_0^2 R^2 + a_2 r_0^4 R^3 + a_3 r_0^6 R^4 \ldots], \tag{7.10}$$

where $a_1, a_2 \ldots$ are numerical constants of order unity. In the special case $a_n = 1$, (7.10) has the form of a geometric series, and one has

$$L_g = \sqrt{-g}\, R/(1 - r_0^2 R). \tag{7.11}$$

The action is here

$$S = \int \sqrt{-g} \left(\frac{R}{1 - r_0^2 R} + \varkappa T \right) \mathrm{d}\Omega, \tag{7.12}$$

and the conditions $\delta S = 0$ would imply that for $T \to \infty$

$$R \to 1/r_0^2 \simeq (m_G/m_p)^2/r_p^2, \tag{7.13}$$

where $m_G c^2 \simeq 10^{16}$ GeV. As a result, there can be no singularity.

8. The Problem of the Coupling Constants

The remaining important problem, why the electromagnetic coupling constant is so much larger than the gravitational coupling, must finally be addressed **.

We begin with the electromagnetic coupling constant. Because electromagnetic waves are associated with a tilting oscillating motion of the vortex rings, it is plausible that the electric charge has its origin in the zero point energy fluctuations tilting the rings (very much as the zero point energy fluctuations of the Planck masses bound in the vortex filaments are responsible for the origin of the gravitational charge). Because electromagnetic waves are described by a vector field equation, the force they transmit must be repulsive.

At the scale of the vortex ring radius r_0, the energy density of the waves is

$$E^2/8\pi \simeq e^2/8\pi r_0^4, \tag{8.1}$$

where e is the value of the electromagnetic coupling constant at the scale r_0. For the zero point fluctuations at the scale r_0 we have according to the uncertainty principle

$$m_G r_0 c \simeq \hbar, \tag{8.2}$$

where $m_G = (r_p/r_0) m_p$. With the volume $(4\pi/3) r_0^3$, this leads to an energy density of the zero point fluctuations which is

$$\varepsilon \simeq (1/2) m_G c^2/(4\pi/3) r_0^3 \simeq \hbar c/8\pi r_0^4. \tag{8.3}$$

** The problem of the coupling constants for the weak and strong interaction can be treated in a similar way [10].

Putting $\varepsilon = E^2/8\pi$ then leads to

$$e^2 \simeq \hbar c, \quad (\text{at} \simeq 10^{16}\,\text{GeV}). \tag{8.4}$$

The extrapolation of the electromagnetic coupling constant from its known value at lower energies to 10^{16} GeV leads to a smaller value than (8.4). Our result is nevertheless in qualitative agreement with the experimental data and quantum electrodynamics, both predicting a rise in the fine structure constant with increasing energy. The likely reason why the electromagnetic coupling constant is smaller than the value given by (8.4) are vacuum polarization effects. They are felt only below the Planck scale, where many Planck masses are involved.

Even though the nondimensional scalar gravitational interaction constant for the Planck masses is $G m_p^2/\hbar c = 1$, this can not be said for the gravitational interaction constant for elementary particles. Since we are restricting ourselves to the electromagnetic and gravitational interactions, the question has to be asked why $G m^2/\hbar c$, where m is the electron mass, is so small. With the value (8.4), $e^2/\hbar c = 1$, taken from our model, one has for the ratio of the gravitational to the electromagnetic interaction

$$G m^2/e^2 = (m/m_p)^2 \cong 1.7 \times 10^{-45}. \tag{8.5}$$

An explanation of this very small number was given previously where it was found that [1]

$$m/m_p = 2\,(r_p/r_0)^6. \tag{8.6}$$

Because $m/m_p \simeq 4.1 \times 10^{-23}$, it would require that

$$r_0/r_p \simeq 7700, \tag{8.7}$$

a value not too far from the value (2.11) derived from the assumption of a minimum energy quantum Reynolds number.

In the Planck aether model, Dirac spinors are excitons made up from a pair of vortex resonances, due to elliptic deformations of the vortex rings, with a mass equal to

$$m_v \simeq \pm\, m_p (r_0/r_p)^2, \quad (m_v c^2 \simeq 10^{12}\,\text{GeV}). \tag{8.8}$$

The positive mass of the Dirac spinor, which is much smaller than m_v, results from the positive (scalar) gravitational interaction energy of the two large masses $\pm\, m_v$ of opposite sign. It can be shown that a superposition of a small mass pole onto a large mass dipole has the property of a Dirac spinor, provided Lorentz invariance acts as a dynamic selection principle.

The very small neutrino mass is more difficult to explain. It could possibly be the result of a compensating gravimagnetic field, superimposed and counteracting the gravistatic field. The gravimagnetic field can reduce the positive mass of the gravistatic field by the factor γ^{-2}, where $\gamma = (1 - v^2/c^2)^{-1/2}$ [15]. According to the uncertainty principle, one would have

$$\gamma\, m_v r_0 c \simeq \hbar \tag{8.9}$$

and hence

$$\gamma \sim r_0/r_p \simeq 8 \times 10^3. \tag{8.10}$$

The neutrino-electron mass ratio would hence be given by

$$m_\nu/m \simeq (r_p/r_0)^2 \simeq 1.6 \times 10^{-8} \tag{8.11}$$

Because the positive and negative mass quasiparticles making up the neutrino would have a large internal rotational motion, the zero point fluctuations from the tilting motion of the vortex rings would here be washed out, which would explain why neutrinos have no electric charge.

9. Conclusion

The Planck aether hypothesis leads to a ground state of the vacuum which can be viewed as a sponge or lattice of quantized vortices. The vortex core radius is equal to the Planck length, with the spacing of the vortex filaments determined by a large universal quantum Reynolds number. With this groundstate two kinds of transverse wave modes, an antisymmetric one connected with a rotational displacement, and a symmetric one with a strain, are possible. The antisymmetric mode can be described by Maxwell's electromagnetic field equations, and the symmetric one by Einstein's gravitational field equations. The forces transmitted by the antisymmetric mode are responsible for holding material objects in a state of static equilibrium, leading to Lorentz contraction as a true physical effect, and to Lorentz invariance as an illusion caused by this true contraction effect. With Lorentz invariance thereby acting as a dynamic selection principle, selecting from all possible states those which are in a static equilibrium, the symmetric wave modes are coupled to the relativistically invariant symmetric energy momentum tensor. Since this energy momentum tensor must include the part resulting from the gravitational field itself, which is represented by the symmetric mode, the field equations become Einstein's nonlinear gravitational field equations. However, because both the electromagnetic and gravitational field equations are derived from the waves of a vortex lattice, the field equations are correct only in the asymptotic limit of dimensions large against the lattice dimensions, and which is the low energy limit. Departures from the asymptotic behavior at low energies, which become increasingly important at high energies, exclude all singular solutions.

The Planck aether hypothesis makes the unification of electromagnetism and gravity almost trivial, because it reduces it to the antisymmetric and symmetric parts for the displacement of the aether. However, because the Planck aether consists of positive and negative Planck masses, it can explain Dirac spinors as well, including the smallness of their gravitational coupling constant.

Symmetry demands an equal number of positive and negative Planck masses in the Planck aether. With all interactions reduced to the zero point fluctuations of the Planck masses, the sum of all charges must be zero, expressed by the symbolic equation

$$\Sigma \,\text{charges} = 0 . \tag{9.1}$$

That the sum of all electric charges is zero is experimentally verified to a high degree of accuracy. Applied to the gravitational charges, (9.1) leads to a vanishing cosmological constant, whereby the (positive) masses in the universe are exactly compensated by the negative gravitational energy. The vanishing of the color charges in QCD is, of course, also covered by theorem (9.1), which in the Planck aether model has the same underlying cause [10].

Appendix

A 1. Avoidance of a Space-Time Singularity with a Nonpolynomial Gravitational Lagrangian

Nonlinear gravitational Lagrangians of the form

$$L_g = \sqrt{-g}\, f(R) \tag{A.I.1}$$

have in the past been considered by Buchdahl [16], resulting in the field equations

$$\begin{aligned} & f'(R)\, R_{ik} - \tfrac{1}{2} g_{ik} f(R) = \varkappa T_{ik}, \\ & (f'(R)\, g^{kl} \sqrt{-g})_{;m} = 0 . \end{aligned} \tag{A.I.2}$$

Contracting the first of these equations results in

$$R f'(R) - 2 f(R) = \varkappa T . \tag{A.I.3}$$

For the special choice

$$f(R) = \frac{R}{1 - r_0^2 R}, \tag{A.I.4}$$

as it was assumed in (7.11), one has for (A.I.3)

$$\frac{R}{(1 - r_0^2 R)^2} - \frac{2R}{1 - r_0^2 R} = \varkappa T . \tag{A.I.5}$$

In the vicinity where $R \to 1/r_0^2$ this can be approximated by

$$\frac{R}{(1 - r_0^2 R)^2} = \varkappa T, \tag{A.I.6}$$

which shows that for $T \to \infty$, $R \to 1/r_0^2$. Therefore, in the limit of an infinite energy density, there can be no space-time singularity.

A 2. Dirac Spinors in the Planck Aether Model

The Dirac spinors result from the gravitational interaction of a mass dipole. The mass dipole is made up from the two masses, one positive and one negative mass, made up from the vortex ring resonance,

$$m_v^{\pm} \simeq \pm\, m_{\mathrm{p}} (r_{\mathrm{p}}/r_0)^2 . \tag{A.II.1}$$

The resonance has the property of quasiparticles. For a positive and a negative mass, the mass of their gravitational interaction is

$$m_0 = \frac{G |m_v^{\pm}|^2}{c^2 r}, \tag{A.II.2}$$

where r is their mutual distance of separation. Because of the positive gravitational interaction energy, the resulting configuration is a mass dipole with a small superimposed mass pole. Putting $m_v^+ + m_0 = m^+$, $m_v^- = m^-$ one has for the center of mass of this pole-dipole configuration of m^+ and m^-

$$m^+ \gamma_+ r_c = |m^-|\, \gamma_- (r_c + r), \tag{A.II.3}$$

where the center of mass located at r_c is on the line connecting m^+ and m^-, but not in between m^+ and m^-. Furthermore,

$$\gamma_+ = (1 - v_+^2/c^2)^{-1/2}, \quad \gamma_- = (1 - v_-^2/c^2)^{-1/2} \tag{A.II.4}$$

with $v_+ = r_c \omega$, $v_- = (r_c + r)\omega$, and with ω the circular angular velocity around r_c. Since $m_0 \leqq m^+ \simeq |m^-|$ one has $r \ll r_c$. For $m^+ > |m^-|$, the positive mass is closer to r_c. Putting $\gamma_+ \equiv \gamma$ one can expand

$$\gamma_- = \gamma\left(1 + \frac{r_c r \omega^2 \gamma^2}{c^2} + \dots\right). \tag{A.II.5}$$

The mass dipole, therefore, is

$$p = m^+ r \simeq |m^-| r = \frac{m^+ \gamma - |m^-| \gamma_-}{\gamma_-} r_c \simeq m_0 r_c/\gamma^2. \tag{A.II.6}$$

For the energy one finds

$$E/c^2 = m = m^+ |\gamma| - |m^-| \gamma_- \simeq p\gamma/r_c \tag{A.II.7}$$

and for the angular momentum

$$J = [m^+ \gamma r_c^2 - |m^-| \gamma_- (r_c + r)^2]\omega \simeq -p\gamma c \simeq -m c r_c. \tag{A.II.8}$$

The Dirac equation has the spin quantization rule

$$J = -(1/2)\hbar; \tag{A.II.9}$$

As we shall see below, under the dynamic selection principle it is the only one possible. It requires that

$$r_c = \hbar/2mc. \tag{A.II.10}$$

From (A.II.6) and (A.II.7) it then follows that

$$m = m_0/\gamma \tag{A.II.11}$$

and hence from (A.II.2)

$$m = \frac{G|m_v^\pm|^2}{c^2 \gamma r}. \tag{A.II.12}$$

With $p \simeq |m_v^\pm| r$, (A.II.7) and (A.II.10) one has

$$2\gamma |m_v^\pm| r c = \hbar. \tag{A.II.13}$$

Eliminating r from (A.II.12) and (A.II.13) results in

$$m = 2G|m_v^\pm|^3/\hbar c = 2|m_v^\pm|^3/m_p^2, \tag{A.II.14}$$

and finally eliminating $|m_v^\pm|$ from (A.II.1) and (A.II.14)

$$m/m_p = 2(r_p/r_0)^6 \tag{A.II.15}$$

To treat this problem within the framework of the Lagrange formalism, one must admit Lagrange functions of the form $L = L(q_k, \dot{q}_k, \ddot{q}_k)$, because even without an external force present, a mass dipole is self-accelerating. To satisfy the dynamic selection principle, Lorentz invariant Lagrange functions must be chosen. The equations of motion follow from

$$\delta \int L(q_k, \dot{q}_k, \ddot{q}_k)\,dt = 0 \tag{A.II.16}$$

with the relativistic four vector

$$u_a = dx_a/ds = \dot{x}_a, \quad ds = \sqrt{1-\beta^2}\,dt, \quad \beta = v/c,$$
$$x_a = (x_1, x_2, x_3, ict) \tag{A.II.17}$$

and with

$$F = u_a^2 = -c^2. \tag{A.II.18}$$

Differentiation to s leads to

$$u_a \dot{u}_a = 0, \quad u_a \ddot{u}_a + \dot{u}_a^2 = 0,$$
$$u_a \dddot{u}_a + 3\dot{u}_a \ddot{u}_a = 0 \dots. \tag{A.II.19}$$

With units for which $c = 1$, (A.II.16) is

$$\delta \int \Lambda(x_a, u_a, \dot{u}_a)\,ds = 0, \tag{A.II.20}$$

where Λ replaces L. With (A.II.18) as subsidiary condition, (A.II.20) gives

$$\frac{d}{ds}\left(\frac{\partial(\Lambda + \lambda F)}{\partial u_a} - \frac{d}{ds}\frac{\partial \Lambda}{\partial \dot{u}_a}\right) - \frac{\partial \Lambda}{\partial x_a} = 0. \tag{A.II.21}$$

Choosing for Λ the linear dependence (k_0, k_1 are constants)

$$\Lambda = -k_0 - \tfrac{1}{2} k_1 \dot{u}_a^2 \tag{A.II.22}$$

one has from (A.II.21)

$$\frac{d}{ds}(2\lambda u_a + k_1 \ddot{u}_a) = 0, \tag{A.II.23}$$

or

$$2\dot{\lambda} u_a + 2\lambda \dot{u}_a + k_1 \dddot{u}_a = 0, \tag{A.II.24}$$

and because of (A.II.19)

$$-2\dot{\lambda} - 3k_1 u_a \ddot{u}_a = -2\dot{\lambda} - \tfrac{3}{2} k_1 \frac{d}{ds}(\dot{u}_a^2) = 0, \tag{A.II.25}$$

hence (summation over v)

$$2\lambda = k_0 - (3/2) k_1 \dot{u}_v^2, \tag{A.II.26}$$

where k_0 appears here as a constant of integration. Inserting (A.II.26) into (A.II.23) the Lagrange multiplier can be eliminated:

$$\frac{d}{ds}[(k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2) u_a + k_1 \ddot{u}_a] = 0. \tag{A.II.27}$$

Writing (A.II.27) as

$$\frac{d\boldsymbol{P}_a}{ds} = 0,$$
$$\boldsymbol{P}_a = (k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2) u_a + k_1 \ddot{u}_a, \tag{A.II.28}$$

one sees that P_a are the components of the energy-momentum four vector. For $k_1 = 0$ one has $P_a = k_0 u_a$ and therefore has to put $k_0 = m$. The mass dipole ist

$$p_a = -k_1 \dot{u}_a \tag{A.II.29}$$

because

$$\frac{\mathrm{d}}{\mathrm{d}s} J_{\alpha\beta} = 0, \tag{A.II.30}$$

where

$$J_{\alpha\beta} = [\boldsymbol{r} \times \boldsymbol{P}]_{\alpha\beta} + [\boldsymbol{p} \times \boldsymbol{u}]_{\alpha\beta} \tag{A.II.31}$$

is the total angular momentum, consisting of an external and an internal (spin) angular momentum part. If $P_k = 0$, $k = 1, 2, 3$, one has only a spin angular momentum which is

$$J_{kl} = [\boldsymbol{p} \times \boldsymbol{u}]_{kl}, \quad k, l = 1, 2, 3. \tag{A.II.32}$$

One furthermore has

$$P_4 = i m = i(k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2)\gamma, \tag{A.II.33}$$

but one has also

$$\begin{aligned} P_a u_a &= -\gamma m = (k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2) u_a^2 + k_1 \ddot{u}_a u_a \\ &= -(k_0 - \tfrac{1}{2} k_1 \dot{u}_v^2). \end{aligned} \tag{A.II.34}$$

One therefore has the double equation

$$m = (k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2)\gamma = (k_0 - \tfrac{1}{2} k_1 \dot{u}_v^2)\gamma^{-1}, \tag{A.II.35}$$

and to keep m finite for $\gamma \to \infty$ one must have $k_0 \to (3/2) k_1 \dot{u}_v^2$ and $k_0 \to \infty$. From (A.II.28), for $P_k = 0$, $k = 1, 2, 3$ follows a radius r_c for the circular motion of the mass dipole with a superimposed mass pole, for which one has

$$p = (k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2) r_c \tag{A.II.36}$$

or because of (A.II.33)

$$p = m r_c / \gamma. \tag{A.II.37}$$

With $\boldsymbol{u} = \gamma \boldsymbol{v}$ one finds

$$J_z = -p u = -m v r_c \simeq -m c r_c, \tag{A.II.38}$$

which is the same as (A.II.8).

For the transition to quantum mechanics one needs the canonical equations of motion. From $\int \Lambda \, \mathrm{d}s = \int L \, \mathrm{d}t$ follows

$$L = \Lambda \sqrt{1 - v^2} \tag{A.II.39}$$

with Λ given by (A.II.22) and where

$$\dot{u}_a^2 = \frac{1}{\sqrt{1 - v^2}^4} \left[\dot{\boldsymbol{v}}^2 + \left(\frac{\boldsymbol{v} \cdot \dot{\boldsymbol{v}}}{\sqrt{1 - v^2}} \right)^2 \right], \tag{A.II.40}$$

therefore $L = L(\boldsymbol{r}, \dot{\boldsymbol{r}}, \ddot{\boldsymbol{r}})$. With

$$\boldsymbol{P} = \frac{\partial L}{\partial \boldsymbol{v}} - \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{\boldsymbol{v}}}, \quad \boldsymbol{\vartheta} = \frac{\partial L}{\partial \dot{\boldsymbol{v}}} \tag{A.II.41}$$

one obtains the Hamilton function

$$H = \boldsymbol{v} \cdot \boldsymbol{P} - \dot{\boldsymbol{v}} \cdot \boldsymbol{\vartheta} - L. \tag{A.II.42}$$

From (A.II.42) follows

$$\begin{aligned} \boldsymbol{\vartheta} &= -\frac{k_1}{\sqrt{1 - v^2}^3} \left[\dot{\boldsymbol{v}} + \frac{(\boldsymbol{v} \cdot \dot{\boldsymbol{v}}) \boldsymbol{v}}{1 - v^2} \right], \\ \dot{\boldsymbol{v}} &= -\frac{\sqrt{1 - v^2}^3}{k_1} [\boldsymbol{\vartheta} - (\boldsymbol{\vartheta} \cdot \boldsymbol{v}) \boldsymbol{v}], \end{aligned} \tag{A.II.43}$$

permitting the elimination of $\dot{\boldsymbol{v}}$ from H. One then finds

$$\begin{aligned} H &= \boldsymbol{v} \cdot \boldsymbol{P} + \sqrt{1 - v^2} \, k_0 \\ &\quad - \frac{1}{2 k_1} \sqrt{1 - v^2}^3 (\boldsymbol{\vartheta}^2 - (\boldsymbol{\vartheta} \cdot \boldsymbol{v})^2). \end{aligned} \tag{A.II.44}$$

H has the same form as the Dirac Hamiltonian if one puts

$$\boldsymbol{P} \to \frac{\hbar}{i} \frac{\partial}{\partial \boldsymbol{r}}, \quad \boldsymbol{v} \to \boldsymbol{a}, \quad \sqrt{1 - v^2} \to a_4, \tag{A.II.45}$$

where $a = (\boldsymbol{a}, a_4)$ are the Dirac matrices, and one obtains

$$\frac{\hbar}{i} \frac{\partial \psi}{\partial t} + H \psi = 0, \tag{A.II.46}$$

$$\begin{aligned} H &= a_1 P_1 + a_2 P_2 + a_3 P_3 + a_4 m, \\ a_\mu^2 &= 1, \quad a_\mu a_\nu + a_\nu a_\mu = 0, \quad \mu \neq \nu, \\ m &= k_0 - \left(\frac{1}{2 k_1} \right) (\boldsymbol{\vartheta}^2 - (\boldsymbol{\vartheta} \cdot \boldsymbol{v})^2). \end{aligned} \tag{A.II.47}$$

The Dirac equation leads to the half-integer spin quantization rule, which was assumed to be true in writing down (A.II.9), but which by requiring the dynamic selection principle under which the Dirac equation was derived, is therefore the only one possible.

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